Review exercise 1

1 a For the first 3 s the cyclist is moving with constant acceleration.

b For the remaining 4 s the cyclist is moving with constant speed.

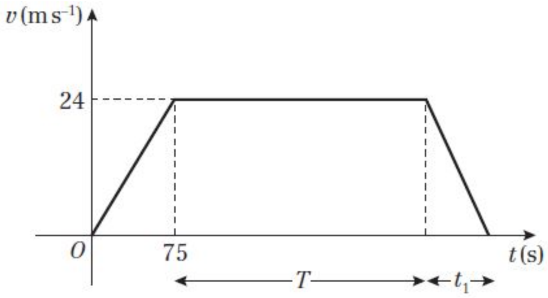
c area = trapezium + rectangle

$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$

$= 10.5 + 20 = 30.5$

The distance travelled by the cyclist is 30.5 m

2 a



b Let time for which the train decelerates be t_1 s.

While decelerating

area $= \frac{1}{2}$ base × height

$600 = \frac{1}{2}t_1 \times 24$

$t_1 = \frac{1200}{24} = 50$

Acceleration is represented by the gradient.

$a = -\frac{24}{t_1} = -\frac{24}{50} = -0.48$

The deceleration is $0.48\,\text{m s}^{-2}$

c For the whole journey

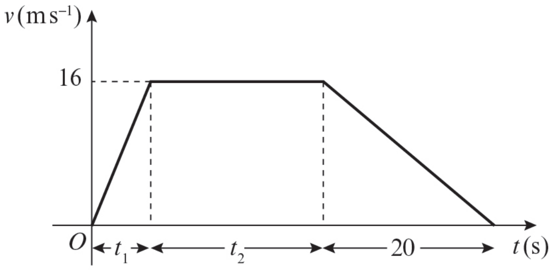
$s = \frac{1}{2}(a+b)h$

$7500 = \frac{1}{2}(T + T + 125)\times 24$

$T = \dfrac{7500 - 1500}{24} = 250$

d Total time is $(75 + T + t_1)\,\text{s} = (75 + 250 + 50)\,\text{s} = 375$ s

3 a



Let the time for which the train accelerates be t_1 s and the time for which it travels at a constant speed be t_2 s.

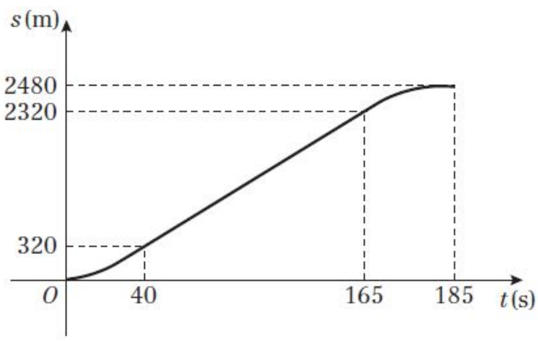
During acceleration
$v = u + at$
$16 = 0 + 0.4t_1 \Rightarrow t_1 = \frac{16}{0.4} = 40$

At constant speed
$2000 = 16 \times t_2 \Rightarrow t_2 = \frac{2000}{16} = 125$

The total time is $(t_1 + t_2 + 20)\,\text{s} = (40 + 125 + 20)\,\text{s} = 185\,\text{s}$

b $s = \frac{1}{2}(a+b)h$
$= \frac{1}{2}(125 + 185) \times 16 = 2\,480$
$AB = 2\,480\,\text{m}$

c



4 a Taking the upwards direction as positive.
$s = 40,\ v = 0,\ a = -9.8,\ u = ?$
$v^2 = u^2 + 2as$
$0^2 = u^2 - 2 \times 9.8 \times 40$
$u^2 = 784 \Rightarrow u = 28$

The speed of projection is 28 m s^{-1}

4 **b** $s = 0,\ u = 28,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 28t - 4.9t^2 = t(28 - 4.9t)$

$t = 0,\ t = \frac{28}{4.9} = 5.714...$

The time taken to return to A is 5.7 s (2 s.f.)

5 Find the speed of projection.
Taking the upwards direction as positive.

$v = 0,\ t = 3,\ a = -9.8,\ u = ?$

$v = u + at$
$0 = u - 9.8 \times 3 \Rightarrow u = 29.4$

$s = 39.2,\ u = 29.4,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$39.2 = 29.4t - 4.9t^2$

$4.9t^2 - 29.4t + 39.2 = 0$

Dividing all terms by 4.9

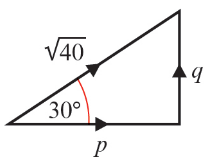
$t^2 - 6t + 8 = 0$

$(t - 2)(t - 4) = 0$

$t = 2, 4$

The ball is 39.2 m above its point of projection when $t = 2$ or when $t = 4$

6



$$\tan 30 = \frac{1}{\sqrt{3}} = \frac{q}{p}$$

$$\therefore q = \frac{p}{\sqrt{3}}$$

$$40 = p^2 + q^2$$

$$p^2 + \frac{p^2}{3} = 40$$

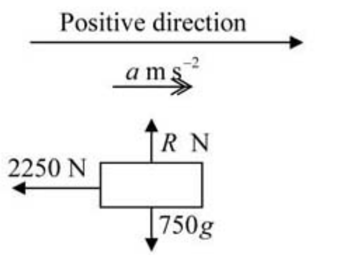
$$\frac{4}{3}p^2 = 40$$

$$p^2 = 30$$

$$\therefore q^2 = 40 - 30 = 10$$

The values are: $p = \sqrt{30}$ and $q = \sqrt{10}$.

7



$F = ma$

$R(\rightarrow)\quad -2250 = 750a$

$a = -\frac{2250}{750} = -3$

$u = 25, v = 15, a = -3, s = ?$

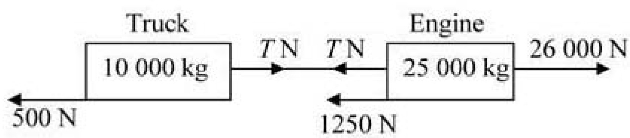
$v^2 = u^2 + 2as$

$15^2 = 25^2 - 6s$

$$s = \frac{25^2 - 15^2}{6} = \frac{400}{6} = 66\tfrac{2}{3}$$

The distance travelled by the car as its speed is reduced is $66\frac{2}{3}$ m.

8



a The resistance on the engine is $25 \times 50 = 1250$ N
The resistance on the truck is $10 \times 50 = 500$ N
For the whole system, the engine and truck

$R(\rightarrow)\qquad F = ma$

$26\,000 - 1250 - 500 = 35\,000a$

$$a = \frac{26\,000 - 1250 - 500}{35\,000} = \frac{97}{140} = 0.6928...$$

The acceleration of the engine and truck is $0.693\,\mathrm{m\,s^{-2}}$ (3 s.f.)

b For the truck alone

$F = ma$

$T - 500 = 10\,000a$

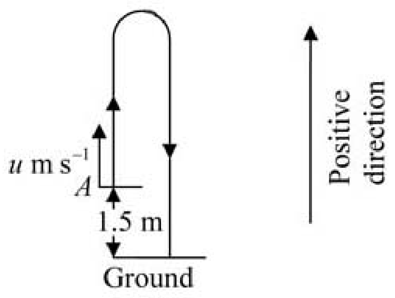
$T = 500 + 10\,000 \times 0.6928... = 7428.57...$

The tension in the coupling is 7430 N (3 s.f.)

c **i** Treating the engine and truck as particles allows us to assume the weight acts from the centre of mass of each object, and ignore wind resistance and rotational forces.

ii By assuming the coupling is a light horizontal rod, we treat it as if it had no mass and therefore can assume it not only stays straight but that it has no weight and the tension is constant along the entire length.

9



a From A to the greatest height, taking upwards as positive.

$v = 0,\ \ a = -9.8,\ \ s = 25.6,\ \ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

b $u = 22.4,\ \ s = -1.5,\ \ a = -9.8,\ \ t = T$

$s = ut + \frac{1}{2}at^2$

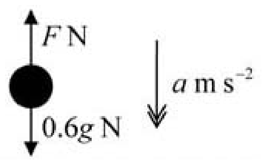
$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

$4.9T^2 - 22.4T - 1.5 = 0$

$$T = \frac{22.4 + \sqrt{(-22.4)^2 - 4 \times 4.9 \times -1.5}}{2 \times 4.9}$$

$= 4.637... = 4.64$ (3 s.f.)

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ \ s = -1.5,\ \ a = -9.8,\ \ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ \ v = 0,\ \ s = 0.025,\ \ a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

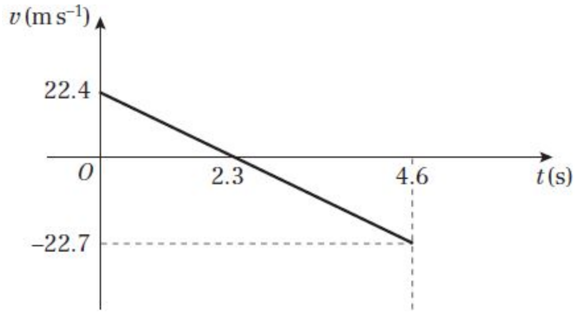
$a = -\frac{531.16}{0.05} = -10\,623.2$

$F = ma$

$0.6g - F = 0.6 \times (-10\,623.2)$

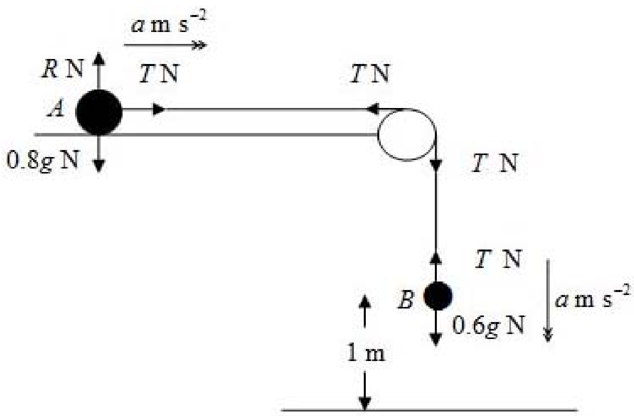
$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.)

9 d



e Consider air resistance during motion under gravity.

10



a For A

$R(\rightarrow)$ $\quad T = 0.8a \quad$ **(1)**

For B

$R(\downarrow)$ $\quad 0.6g - T = 0.6a \quad$ **(2)**

(1) + **(2)**

$0.6g = 1.4a$

$$a = \frac{0.6 \times 9.8}{1.4} = 4.2$$

The acceleration of A is 4.2 m s^{-2}

b Substitute $a = 4.2$ into (1)

$T = 0.8 \times 4.2 = 3.36$

The tension in the string is 3.4 N (2 s.f.)

c $u = 0, \ a = 4.2, \ s = 1, \ v = ?$

$v^2 = u^2 + 2as$

$\quad = 0^2 + 2 \times 4.2 \times 1 = 8.4$

$v = \sqrt{8.4} = 2.898...$

The speed of B when it hits the ground is 2.9 m s^{-1} (2 s.f.)

10 d $u = 0,\ a = 4.2,\ s = 1,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$1 = 0 + 2.1t^2$

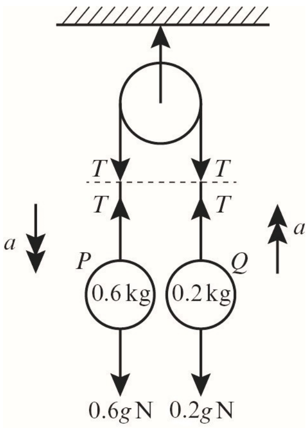
$t^2 = \frac{1}{2.1} \Rightarrow t = 0.690...$

The time taken for B to reach the ground is 0.69 s (2 s.f.)

e **i** Describing the string as 'light' means it has no mass (and therefore no weight).

ii This fact allows us to assume that the tension is constant in all parts of the string.

11



a $F = ma$
For P ↓ positive: $0.6a = 0.6g - T$ **(1)**
For Q ↑ positive: $0.2a = T - 0.2g$ **(2)**
$3 \times$ (**2**): $0.6a = 3T - 0.6g$
Subtracting (**1**) from this: $0 = 4T - 1.2g$
$4T = 1.2g = 1.2 \times 9.8$
The tension in the string is 2.9 N (2 s.f.)

b (1) + (2): $0.8a = 0.4g$
$a = \frac{g}{2} = \frac{9.8}{2}$
The acceleration is 4.9 m s^{-2}.

11 c For P, before string breaks, taking up as positive:
$s = ?$, $u = 0$, $a = 4.9$, $t = 0.4$

$$s = ut + \tfrac{1}{2}at^2$$
$$s = (0 \times 0.4) + \tfrac{1}{2}(4.9 \times 0.4^2)$$
$$= \tfrac{1}{2}(0.784)$$
$$= 0.392\,\text{m}$$

The total distance P has to fall is therefore $1 - 0.392 = 0.608$ m.

$v = u + at$
$v = 0 + (0.4 \times 4.9) = 1.96\ \text{m s}^{-1}$
Before the string breaks, P is moving downwards at $1.96\ \text{m s}^{-1}$. After string breaks, taking down as positive, P moves under gravity.

$s = 0.608$, $u = 1.96$, $a = 9.8$, $t = ?$

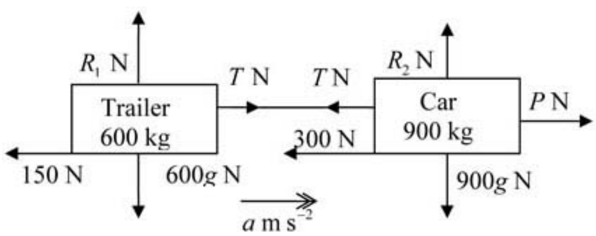
$$s = ut + \tfrac{1}{2}at^2$$
$$0.608 = 1.96t + \tfrac{1}{2}(9.8 \times t^2)$$
$$0 = 4.9t^2 - 1.96t - 0.608$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-1.96 \pm \sqrt{(-1.96)^2 - (4 \times 4.9 \times -0.608)}}{2 \times 4.9}$$
$$= \frac{-1.96 \pm \sqrt{15.76}}{9.8}$$
$$= 0.205\,\text{s or} -0.605\,\text{s (3 d.p.)}$$

Only positive answers are relevant in this context. $\therefore$ P hits the floor 0.21 s (2 s.f.) after the string breaks.

d This fact allows us to assume that the tension is constant in all parts of the string and that the acceleration of the two particles is the same.

12



a i For the whole system:

$$F = ma$$
$$R(\rightarrow) \qquad P - 300 - 150 = 1500 \times 0.4$$
$$P = 1050$$

The tractive force exerted by the engine of the car is 1050 N.

12 a ii For the trailer alone:

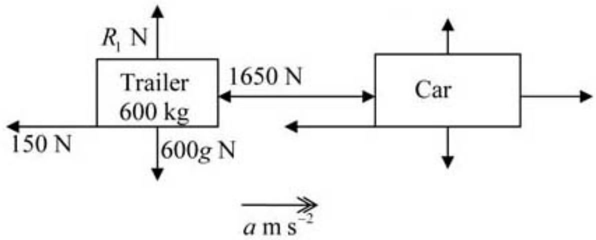
$$F = ma$$

$$R(\rightarrow) \qquad T - 150 = 600 \times 0.4$$

$$T = 390$$

The tension in the tow bar is 390 N.

b For the trailer alone:



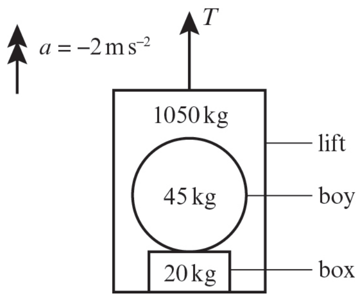
$$F = ma$$

$$R(\rightarrow) \qquad 1650 - 150 = 600a$$

$$a = -\tfrac{1500}{600} = -2.5$$

The greatest possible deceleration of the car is 2.5 m s^{-2}

13

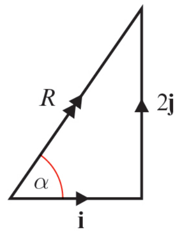


a $F = ma$
Taking up as positive:
$(1050 + 45 + 20) \times -2 = T - (1050 + 45 + 20)g$
$T = 1115(g - 2)$
$T = 1115 \times 7.8$
The tension in the cable is 8697 N.

b From Newton's third law of motion:
|Force exerted on boy by box| = |Force exerted on box by boy| = $|R_1|$
For the boy, taking up as positive:
$45 \times -2 = R_1 - 45g$
$R_1 = 45(g - 2)$
$R_1 = 45 \times 7.8$
The boy exerts a force of 351 N on the box.

13 c From Newton's third law of motion:
|Force exerted on box by lift| = |Force exerted on lift by box| = $|R_2|$
For the box, taking up as positive:
$20 \times -2 = R_2 - 20g - 351$
$R_2 = 351 + 20(g - 2)$
$R_2 = 351 + (20 \times 7.8) = 351 + 156$
The box exerts a force of 507 N on the lift.

14 a



Let the required angle be α.
Then $\tan\alpha = 2$
$\therefore \alpha = 63^{\circ}\,(2\text{ s.f.})$

b As $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{R}$
$(2\mathbf{i} + 3\mathbf{j}) + (\lambda\mathbf{i} + \mu\mathbf{j}) = k(\mathbf{i} + 2\mathbf{j})$

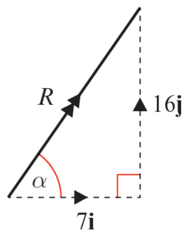
where k is a constant.
$\therefore 2 + \lambda = k$ and $3 + \mu = 2k$ *

Eliminate k from these two equations.
Then $2(2+\lambda) = 3 + \mu$
$\therefore 2\lambda - \mu + 1 = 0$

c If $\mathbf{F}_2$ is parallel to $\mathbf{j}$ then $\lambda = 0$
Substituting $\lambda = 0$ into * gives

$\mu = 1$ and $k = 2$
$\therefore \mathbf{R} = 2\mathbf{i} + 4\mathbf{j}$
$\therefore |\mathbf{R}| = \sqrt{2^2 + 4^2}$
$= \sqrt{20}$
$= 4.47$ (3 s.f.)

15



a The magnitude of

$$\mathbf{R} = \sqrt{7^2 + 16^2}$$
$$= 17.5 \text{ (1 d.p.)}$$

b $\tan\alpha = \frac{16}{7}$

$$\alpha = \tan^{-1}\left(\tfrac{16}{7}\right)$$
$$= 66^\circ \text{ (nearest degree)}$$

c Let $\mathbf{P} = \lambda(\mathbf{i} + 4\mathbf{j})$ and $\mathbf{Q} = \mu(\mathbf{i} + \mathbf{j})$

As $\mathbf{P} + \mathbf{Q} = \mathbf{R}$

$\therefore \lambda(\mathbf{i} + 4\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j}) = (7\mathbf{i} + 16\mathbf{j})$

Equating **i** components

$\lambda + \mu = 7$ **(1)**

Equating **j** components

$4\lambda + \mu = 16$ **(2)**

Subtract **(2)** − **(1)**

$3\lambda = 9$

$\therefore \lambda = 3$

Substitute into equation **(1)**

$\therefore 3 + \mu = 7$

$\therefore \mu = 4$

$\therefore \mathbf{P} = 3(\mathbf{i} + 4\mathbf{j}) = 3\mathbf{i} + 12\mathbf{j}$ and $\mathbf{Q} = 4(\mathbf{i} + \mathbf{j}) = 4\mathbf{i} + 4\mathbf{j}$

16 Rich has position vector $\mathbf{i}-6\mathbf{j}$.
Dev has position vector $\mathbf{9i}+2\mathbf{j}$.

a Dev's position relative to Rich's position is
$9\mathbf{i}+2\mathbf{j}-(\mathbf{i}-6\mathbf{j})=8\mathbf{i}+8\mathbf{j}$
The distance d between Rich and Dev is
$d=\sqrt{8^2+8^2}$
$=8\sqrt{2}$ km
$=11.3$ km (3 s.f.)

b $\mathbf{r}_R=\mathbf{i}-6\mathbf{j}+t(\mathbf{i}+6\mathbf{j})$
$\mathbf{r}_D=\mathbf{9i}+2\mathbf{j}+t(-3\mathbf{i}+2\mathbf{j})$
If Rich and Dev meet at time t,
$\mathbf{i}-6\mathbf{j}+t(\mathbf{i}+6\mathbf{j})=\mathbf{9i}+2\mathbf{j}+t(-3\mathbf{i}+2\mathbf{j})$
$4t\mathbf{i}+4t\mathbf{j}=8\mathbf{i}+8\mathbf{j}$
This will be true when $4t=8$
$t=2$ hours
Therefore, they meet at 14:00.

c Substituting $t=2$ into the equation for r_R (equation for r_D could be used instead) gives meeting point:
$\mathbf{r}=\mathbf{i}-6\mathbf{j}+2(\mathbf{i}+6\mathbf{j})$
$=\mathbf{i}-6\mathbf{j}+2\mathbf{i}+12\mathbf{j}$
$=(3\mathbf{i}+6\mathbf{j})$ km

17 a $\mathbf{R}=2\mathbf{i}-5\mathbf{j}+\mathbf{i}+\mathbf{j}$
$=3\mathbf{i}-4\mathbf{j}$
$|\mathbf{R}|=\sqrt{3^2+4^2}=\sqrt{25}=5$ N

b $\mathbf{R}_{\text{new}}=3\mathbf{i}-4\mathbf{j}+k\mathbf{j}$
$\tan 45°=1$, so the coefficients of **i** and **j** are equal.
So $-4\mathbf{j}+k\mathbf{j}=3\mathbf{i}$
So $k=7$

18 $\sin 60°=\dfrac{m}{100}$
$m=100\sin 60°$
$=50\sqrt{3}$
Using Pythagoras' theorem:
$n=\sqrt{100^2-\left(50\sqrt{3}\right)^2}+30=\sqrt{2500}+30=50+30=80$
or using $n=100\cos 60°+30=80$

19 a Call the finish line F:

$\overrightarrow{AF} = -65\mathbf{i} + 180\mathbf{j} - 10\mathbf{i} = -75\mathbf{i} + 180\mathbf{j}$

$AF = \sqrt{75^2 + 80^2} = \sqrt{38025} = 195$

$\overrightarrow{BF} = 100\mathbf{i} + 120\mathbf{j} - 10\mathbf{i} = 90\mathbf{i} + 120\mathbf{j}$

$BF = \sqrt{90^2 + 120^2} = \sqrt{22500} = 150$

150 < 195, so boat B is closer to the finish line.

b Speed of boat A $= \sqrt{2.5^2 + 6^2}$
$= \sqrt{42.25}$
$= 6.5$ m/s

Speed of boat B $= \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$ m/s

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

Time taken for boat A to reach the finish line $= \dfrac{195}{6.5} = 30$ s

Time taken for boat B to reach the finish line $= \dfrac{150}{5} = 30$ s

Both boats reach the finish line at the same time.

Challenge

1 $t_1 + t_2 + t_3 = 7 \times 60 = 420$
$3t_1 = 4t_3$
$t_3 = 0.75t_1$

Considering time t_1

$$s = \left(\frac{u+v}{2}\right)t_1$$

$$1750 = \left(\frac{0+v}{2}\right)t_1$$

$$v = \frac{3500}{t_1}$$

Considering time t_2
$s_2 = vt_2$

$$17\,500 = \frac{3500}{t_1}t_2$$

$$t_2 = 5t_1$$

Considering total time:
$t_1 + 5t_1 + 0.75\ t_1 = 420$

$$t_1 = \frac{420}{6.75} = 62.22\ \text{s}$$

$\therefore t_2 = 311.11$ s
& $t_3 = 46.67$ s

Distance travelled during time t_3 is s_3

$$s_3 = \left(\frac{u+v}{2}\right)t_3$$

$$u = \frac{3500}{t_1} = \frac{3500}{62.22} = 56.25\text{ , } v = 0,\ t = 46.67$$

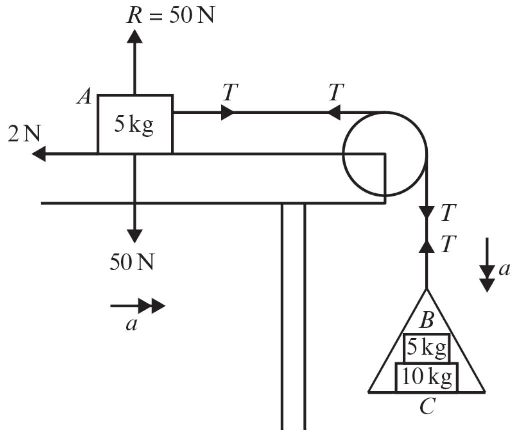
$$s_3 = \left(\frac{56.25+0}{2}\right)46.67$$

$$s_3 = 28.125 \times 46.67 = 1312.6$$

Total distance = 1750 + 17 500 + 1312.6
The distance between the two stations is 20.6 km (3 s.f.).

Challenge

2



a Considering A, → positive: $T - 2 = 5a$
Considering entire pan, ↓ positive: $(5 + 10)g - T = 15a$
So $150 - T = 15a$

Adding these gives:
$148 = 20a$
The acceleration of the pan is 7.4 m s^{-2}.

b Substituting this value into the first equation gives:
$T - 2 = 5 \times 7.4 = 37$
The tension in the string is 39 N.

c Block C exerts a normal reaction force R on block B.
Considering block B only, ↓ positive:
$5g - R = 5a$
$50 - R = 37$
Block C exerts a force of 13 N on block B.

d The force the string exerts on the pulley has two perpendicular components, each of magnitude T.
The magnitude of the total force, F, is therefore given by:
$F^2 = T^2 + T^2$
$F = \sqrt{39^2 + 39^2} = \sqrt{3042}$
The string exerts a force of magnitude 55 N (2 s.f.) on the pulley.

e The fact that the string is inextensible allows us to assume that the tension is constant in all parts of the string and that the acceleration of Block A and the scale pan are the same.

16 Rich has position vector $\mathbf{i}-6\mathbf{j}$
Dev has position vector $\mathbf{9i}+2\mathbf{j}$

a Dev's position relative to Rich's position is
$9\mathbf{i}+2\mathbf{j}-(\mathbf{i}-6\mathbf{j})=8\mathbf{i}+8\mathbf{j}$
The distance d between Rich and Dev is
$d=\sqrt{82+82}$
$=\sqrt{128}$
$=8\sqrt{2}$ km

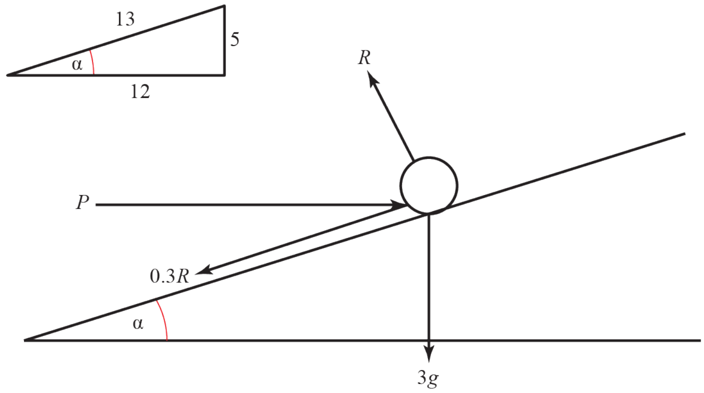
b $\mathbf{i}-6\mathbf{j}+t(\mathbf{i}+6\mathbf{j})=\mathbf{9i}+2\mathbf{j}+t(-3\mathbf{i}+2\mathbf{j})$
$4t\mathbf{i}+4t\mathbf{j}=8\mathbf{i}+8\mathbf{j}$
Comparing coefficients for $\mathbf{i}$
$4t=8$
$t=2$ hours
Therefore they meet at 2 pm

c They meet at
$\mathbf{r}=\mathbf{i}-6\mathbf{j}+2(\mathbf{i}+6\mathbf{j})$
$=\mathbf{i}-6\mathbf{j}+2\mathbf{i}+12\mathbf{j}$
$=(3\mathbf{i}+6\mathbf{j})$ km

Review exercise 2

1 Since the particle is moving at constant velocity, the forces acting on it are balanced.

$\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$



$R(\nwarrow)$:

$R = 3g\cos\alpha + P\sin\alpha$

$R = \frac{3g \times 12}{13} + \frac{5P}{13}$

$R = \frac{36g + 5P}{13}$

$R(\nearrow)$:

$P\cos\alpha = \mu R + 3g\sin\alpha$

$\frac{12}{13}P = \frac{1}{5}\left(\frac{36g+5P}{13}\right) + \frac{3g\times 5}{13}$

$12P = \frac{36g}{5} + P + 15g$

$11P = \left(\frac{36}{5} + 15\right)g$

$P = \left(\frac{36}{5} + 15\right) \times \frac{9.8}{11}$

$= 19.778...$

P is 19.8 N (to 3 s.f.).

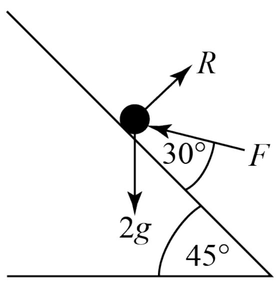
2 Using Newton's 2nd Law, $F = ma$ in the direction of the acceleration

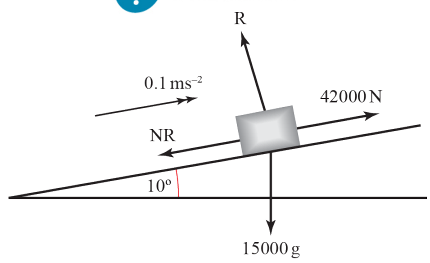
$F\cos 30° - 2g\cos 45° = 2\times 2$

$\frac{\sqrt{3}}{2}F - \sqrt{2}g = 4$

$F = \frac{2}{\sqrt{3}}\left(4 + \sqrt{2}g\right)$

$= 20.6$ N (3 s.f.)





3 $m = 15\,000$ kg, $a = 0.1$ ms^{-2}

a $R(\nwarrow)$:

$R = 15\,000g\cos 10^\circ$

$R = 15\,000 \times 9.8\cos 10^\circ = 144\,767$

To the nearest whole newton, the reaction between the container and the slope is 144 767 N.

b Using Newton's second law of motion and resolving up the slope:

$$F = ma$$

$$42\,000 - \mu R - 15\,000g\sin 10^\circ = 15\,000 \times 0.1$$

$$\mu \times 144\,767 = 42\,000 - 1500 - \left(15\,000 \times 9.8\sin 10^\circ\right)$$

$$\mu = \frac{40\,500 - 25\,526.2}{144\,767}$$

$$= 0.103\,433...$$

The coefficient of friction between the container and the slope is 0.103 (3 s.f.).

c Using Newton's second law of motion and resolving down the slope after winch stops working:

$$F = ma$$

$$\mu R + 15\,000g\sin 10^\circ = 15\,000a$$

$$144\,767 \times 0.103\,433 + 15\,000g\sin 10^\circ = 15\,000a \quad \text{(using results from } \mathbf{a} \text{ and } \mathbf{b}\text{)}$$

$$a = \frac{40\,500}{15\,000}$$

$$= 2.7$$

So the container accelerates down the slope at 2.7 ms^{-2}

So: $u = -2$ ms^{-1}, $v = 0$ ms^{-1}, $a = 2.7$ ms^{-2}, $t = ?$

$v = u + at$

$0 = -2 + 2.7t$

$t = \frac{2}{2.7}$

$= 0.74074...$

The container takes 0.740 s (3 s.f.) to come to rest.

d Once the container comes to rest, the container will tend to move down the slope and hence the frictional force will act up the slope. The container will therefore move back down if the component of weight down the slope is greater than the frictional force; i.e. if

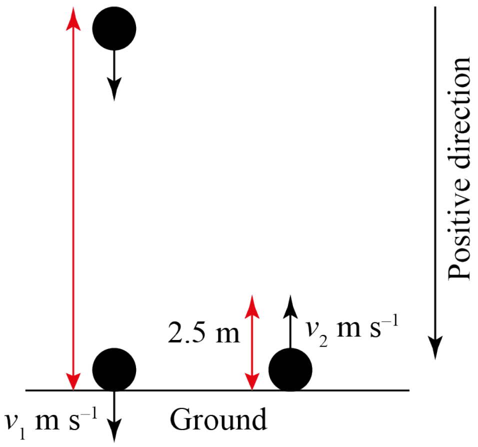
$$mg\sin 10^\circ > \mu R$$

$$15\,000g\sin 10^\circ > 144\,767 \times 0.103\,433$$

$$25\,526 > 14\,974$$

Since this inequality is true, the container will start to slide back down the slope.

4



As ball descends

$u = 0, a = 9.8, s = 10, v = v_1$

$v^2 = u^2 + 2as$

$v_1^2 = 0^2 + 2 \times 10 \times 9.8 = 196$

$v_1 = \sqrt{196} = 14$

The ball is released from rest 10 m above the ground. The first step is to calculate the speed with which the ball strikes the ground.

4 After rebound

$v = 0, a = 9.8, s = -2.5, u = v_2$

$v^2 = u^2 + 2as$

$0^2 = v_2^2 + 2 \times 9.8 \times (-2.5) \Rightarrow v_2^2 = 49$

$v_2 = -\sqrt{49} = -7$

$I = mv_2 - mv_1$

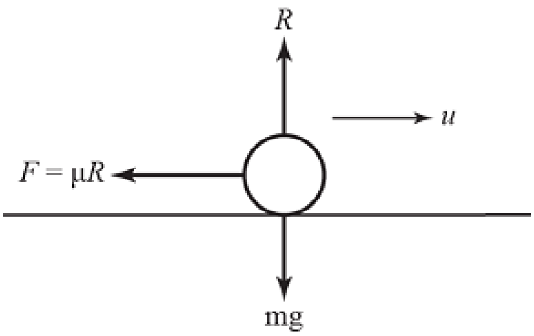
$= 0.3 \times (-7) - 0.3 \times 14 = -6.3$

You must then use the fact that the ball reaches a maximum height of 2.5 m to find the velocity with which it rebounds from the ground.

As it rebounds from the ground, the ball is moving upwards. That is in the negative direction. You must take the negative square root of 49, which is –7.

The magnitude of the impulse is 6.3 N.

5 a $m = 0.250$ kg, $mu = 2$ Ns, $\mu = 0.2$, $v = 0$ ms^{-1}, $s = ?$



Resolving vertically:
$R = mg$

Friction is limiting, so $F = \mu R = \mu mg$

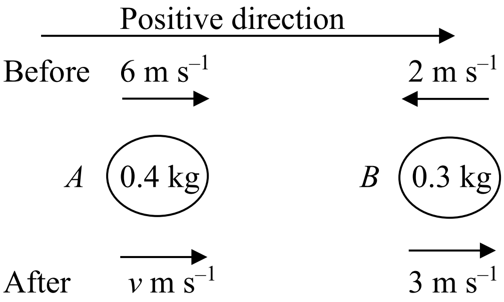
Impulse on car = change of momentum of car:

$$Ft = mv - mu$$
$$\mu mgt = 0 - (-2)$$
$$t = \frac{2}{\mu mg}$$
$$s = \tfrac{1}{2}(u + v)t$$
$$s = \frac{1}{2}\left(\frac{2}{m} + 0\right)\frac{2}{\mu mg}$$
$$s = \frac{2}{\mu m^2 g} = \frac{2}{0.2 \times 0.25^2 \times 9.8} = 16.3265...$$

The racing car travels a distance of 16 m (2 s.f.) past point A before coming to a stop.

b The car stops in a shorter distance because there will be additional frictional forces acting on it (e.g. air resistance) which will increase the deceleration.

6



The total linear momentum before impact must equal the total linear momentum after impact.

a Conservation of linear momentum

The velocity of B before impact is in the negative direction so it must be entered as –2 in any equations involving linear momentum.

$$0.4\times6+0.3\times(-2)=0.4\times v+0.3\times3$$
$$2.4-0.6=0.4v+0.9$$
$$0.4v=2.4-0.6-0.9=0.9$$
$$v=\frac{0.9}{0.4}=2.25$$

The velocity of A is positive (2.25 m s^{-1}) after impact and it was positive (6 m s^{-1}) before impact. So the direction of motion of A is unchanged.

The speed of A after the collision is 2.25 m s^{-1}

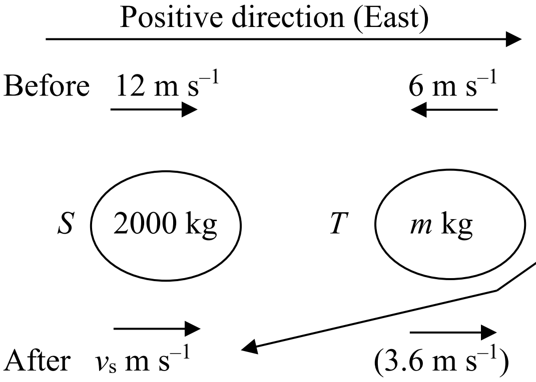
The direction of motion of A is unchanged.

b For $B, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=0.3\times3-0.3\times(-2)$$
$$=0.9+0.6=1.5$$

The magnitude of the impulse exerted on B is 1.5 N s

7



You do not know which direction S will be moving in after the impact. Mark the unknown velocity as v m s^{-1} in the positive direction. After you have worked out v, the sign of v will tell you the direction in which S is moving.

a For $S, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$-28800=2000\times v_s-2000\times12$$
$$2000v_s=-28800+24000=-4800$$
$$v_s=-\frac{4800}{2000}=-2.4$$

The speed of S immediately after the collision is 2.4 m s^{-1}

The sign of v is negative, so S is moving in the negative direction. In this solution, the positive direction has been taken as east, so S is now moving west.

b Immediately after the collision S is moving due west.

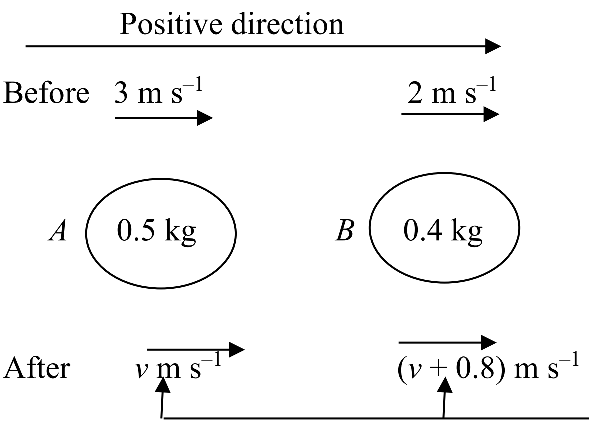
7 c Conservation of linear momentum

$$2000\times 12+m\times(-6)=2000\times(-2.4)+m\times 3.6$$

$$9.6m=24\,000+4800=28\,800 \Rightarrow m=\frac{28\,800}{9.6}=3000$$

The mass of T is 3000 kg

8



You need to translate the statement that 'the speed of B is 0.8 m s^{-1} greater than the speed of A' into algebra. If the speed of A after the collision is v m s^{-1} then the speed of B is 0.8 m s^{-1} greater; that is $(v + 0.8)$m s^{-1}

a Conservation of linear momentum

$$0.5\times 3+0.4\times 2=0.5\times v+0.4(v+0.8)$$
$$1.5+0.8=0.5v+0.4v+0.32$$
$$0.9v=1.5+0.8-0.32=1.98$$
$$v=\frac{1.98}{0.9}=2.2$$

All velocities in this part are in the positive direction.

The speed of A after the collision is 2.2 m s^{-1}

The speed of B after the collision is $(2.2 + 0.8)$ m s^{-1} = 3 m s^{-1}

To find the speed of B add 0.8 m s^{-1} to the speed of A.

b The momentum of A before the collision is given by

$$mu=0.5\times 3\,\text{N s}=1.5\,\text{N s}$$

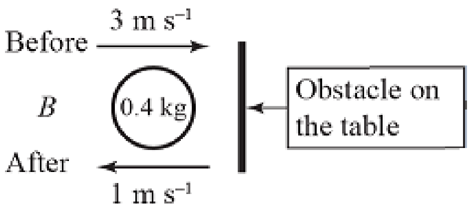
The momentum of a particle is its mass times its velocity. Momentum is a vector quantity.

The momentum of A after the collision is given by

$$mv=0.5\times 2.2\,\text{N s}=1.1\,\text{N s}$$

A loses a momentum of $(1.5-1.1)\,\text{N s}=0.4\,\text{N s}$, as required.

8 c



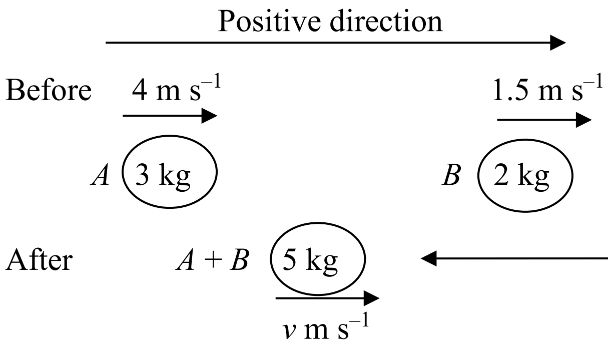
For B, before and after the second impact

$$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$$
$$= 0.4 \times (-1) - 0.4 \times 3$$
$$= -1.6$$

Left to right has been taken as the positive direction throughout the question. The impulse on B is negative as, as the situation is drawn here, the impulse on B is in the direction from right to left.

The magnitude of the impulse received by B in this second impact is 1.6 Ns

9



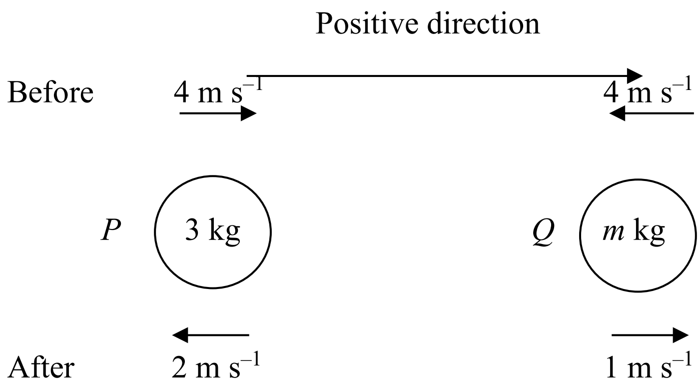
After the collision A (of mass 3 kg) and B (of mass 2 kg) combine to form a single particle. That particle will have the mass which is the sum of the two individual masses, 5 kg.

Conservation of linear momentum

$$4 \times 3 + 2 \times 1.5 = 5 \times v$$
$$12 + 3 = 5v \Rightarrow v = \frac{15}{5} = 3$$

The speed of C immediately after the collision is 3 m s^{-1}

10



a Conservation of linear momentum

$$3\times4+m\times(-4)=3\times(-2)+m\times1$$
$$12-4m=-6+m\Rightarrow 5m=18$$
$$m=\frac{18}{5}=3.6$$

> In the equation for the conservation of momentum, you must give the velocities in the negative direction a negative sign.

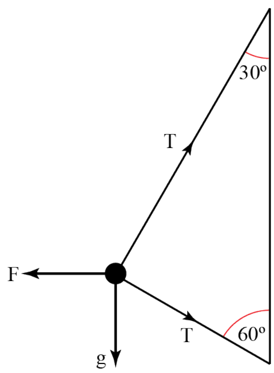
b For $Q, \mathbf{I}=m\mathbf{v}-m\mathbf{u}$

$$I=3.6\times1-3.6\times(-4)$$
$$=3.6+14.4=18$$

The magnitude of the impulse exerted on Q in the collision is 18 N s

> As the magnitude of the impulse exerted on P is the same as the magnitude of the impulse exerted on Q, you could equally correctly work out the change in linear momentum of P. The working then would be $I=3\times(-2)-3\times4=-18$, which gives the same magnitude, 18 Ns

11 The system is in equilibrium.



a Resolving vertically:

$$T\cos60^\circ+g=T\cos30^\circ$$
$$\frac{T}{2}+g=\frac{T\sqrt{3}}{2}$$
$$2g=T\left(\sqrt{3}-1\right)$$
$$T=\frac{2g}{\sqrt{3}-1}\quad\text{as required.}$$

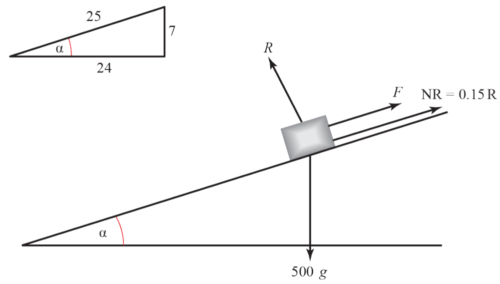
b Resolving horizontally:

$$F=T\sin60^\circ+T\sin30^\circ$$
$$F=T\left(\sin60^\circ+\sin30^\circ\right)$$
$$F=\left(\frac{2g}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{2}\right)$$
$$F=\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)g$$

c We model the bead as smooth in order to assume there is no friction between it and the string.

12 $\tan\alpha = \frac{7}{24} \Rightarrow \sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$

The system is in equilibrium.



a $R(\nwarrow)$:

$R = 500g\cos\alpha$

$R = \frac{24}{25}\times 500g = 480g$

The normal reaction of the hill on the crate is $480g$ N, as required.

b Minimum value of F occurs when the crate is on the point of sliding down the hill. Frictional force then acts up the hill.

$R(\nearrow)$:

$F + \mu R = 500g\sin\alpha$

$F = \left(\frac{7}{25}\times 500g\right) - \left(\frac{3}{20}\times 480g\right)$ (Using $\mu = \frac{3}{20}$, and $R = 480g$ from **a**)

$F = (140 - 72)g$

$= 68g$

The minimum value of F required to maintain equilibrium is $68g$ N.

13 For the 3 kg mass

R(↑) $T = 3g$ **(1)**

For the 12 kg mass

R(→) $T = \mu R$ **(2)**

R(↑) $R = 12g$ **(3)**

Equating **(1)** and **(2)** gives

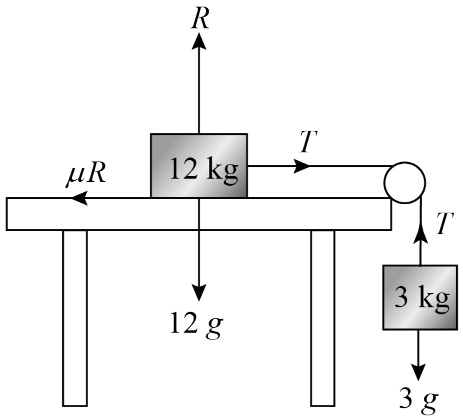
$\mu R = 3g$

$\mu = \frac{3g}{R}$

Then substituting $R = 12g$ gives

$\mu = \frac{3g}{12g}$

$= 0.25$



14 Assuming that the box of mass m_1 is on the point of sliding down the slope and the box of mass m_2 is on the point of sliding up the slope.

For box of mass m_1

R(↗) $T + \mu R_1 = m_1 g\sin\theta_1$

R(↖) $R_1 = m_1 g\cos\theta_1$

So $T = m_1 g\sin\theta_1 - \mu m_1 g\cos\theta_1$ **(1)**

For box of mass m_2

R(↖) $T = m_2 g\sin\theta_2 + \mu R_2$

R(↗) $R_2 = m_2 g\cos\theta_2$

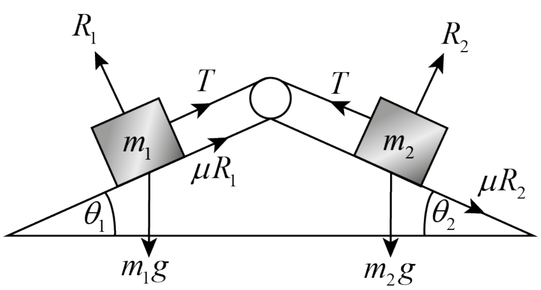
So $T = m_2 g\sin\theta_2 - \mu m_2 g\cos\theta_2$ **(2)**

Equating **(1)** and **(2)** gives

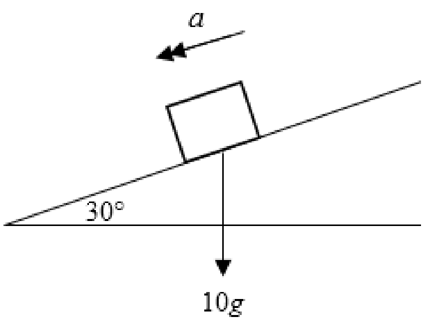
$m_1 g\sin\theta_1 - \mu m_1 g\cos\theta_1 = m_2 g\sin\theta_2 + \mu m_2 g\cos\theta_2$

$m_1 g\sin\theta_1 - m_2 g\sin\theta_2 = \mu m_1 g\cos\theta_1 + \mu m_2 g\cos\theta_2$

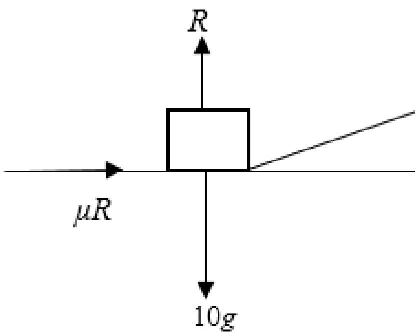
$\mu = \frac{m_1\sin\theta_1 - m_2\sin\theta_2}{m_1\cos\theta_1 + m_2\cos\theta_2}$



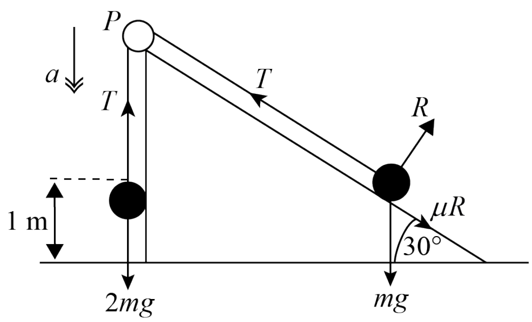
15 a (↙) Newton's 2nd law, $F = ma$
$10g \sin 30 = 10a$
$a = 0.5g$
$u = 0$, $a = 0.5g$ and $s = 10$
Using $v^2 = u^2 + 2as$ gives
$v^2 = 2(0.5g)(10)$
$\quad = 10g$
$v = 7\sqrt{2}$ m s^{-1}
$\quad = 9.90$ m s^{-1} (3 s.f.)



b R(↑) $R = 10g$
(←) Newton's 2nd law, $F = ma$
$-\mu R = 10a$
$10a = -0.2(10g)$
$a = -0.2g = -1.96$ m s^{-2}
Using $v^2 = u^2 + 2as$ gives
When the box comes to rest $v = 0$, so
$0 = 10g - 2(0.2g)s$
$s = \dfrac{10g}{0.4g}$
$\quad = 25$
So the total distance travelled by the box is 10 m + 25 m = 35 m



16 a For the $2m$ mass
(↓) Newton's 2nd law, $F = ma$
$2mg - T = 2ma$
$T = 2mg - 2ma$ **(1)**
For the m mass
R(↗) $R = mg \cos 30$
$= \dfrac{\sqrt{3}}{2}mg$
(↖) Newton's 2nd law, $F = ma$
$T - mg \sin 30 - \mu R = ma$
Since $\mu = \dfrac{1}{5}$
$T - \dfrac{1}{2}mg - \dfrac{1}{5}\left(\dfrac{\sqrt{3}}{2}mg\right) = ma$
$T = ma + \dfrac{5+\sqrt{3}}{10}mg$ **(2)**
Equating **(1)** and **(2)** gives
$2mg - 2ma = ma + \dfrac{5+\sqrt{3}}{10}mg$
$3a = 2g - \dfrac{5+\sqrt{3}}{10}g$
$a = \dfrac{15-\sqrt{3}}{30}g$
$\quad = 4.33$ m s^{-2} (3 s.f.)



16 b To find speed of A at the point B hits the ground:

$u = 0$, $s = 1$ and $a = \dfrac{15-\sqrt{3}}{30}g$

using $v^2 = u^2 + 2as$ gives

$$v^2 = 2\left(\frac{15-\sqrt{3}}{30}g\right)$$
$$= \frac{15-\sqrt{3}}{15}g$$

Once B has hit the ground, there will be no tension in the string.

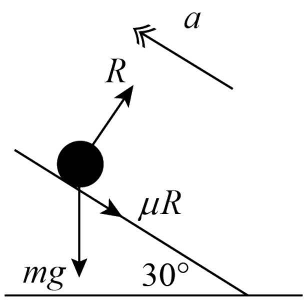
R(↗) $R = mg\cos 30$

$$= \frac{\sqrt{3}}{2}mg$$

(↘) Newton's 2nd law, $F = ma$

$Mg\sin 30 + \mu R = -ma$

Since $\mu = \dfrac{1}{5}$



$$\frac{1}{2}mg + \frac{1}{5}\left(\frac{\sqrt{3}}{2}mg\right) = -ma$$

$$a = -\left(\frac{1}{2}g + \frac{1}{5}\left(\frac{\sqrt{3}}{2}g\right)\right)$$
$$= -\left(\frac{5+\sqrt{3}}{10}\right)g$$

To find the distance travelled by A after B hits the ground, use $v^2 = u^2 + 2as$

$$0^2 = \left(\frac{15-\sqrt{3}}{15}\right)g - 2\left(\frac{5+\sqrt{3}}{10}\right)gs$$

$$\left(\frac{5+\sqrt{3}}{5}\right)s = \left(\frac{15-\sqrt{3}}{15}\right)$$

$$s = \frac{\left(15-\sqrt{3}\right)}{3\left(5+\sqrt{3}\right)}$$

Finally add on the 1 m that A travelled before B hit the ground to find the total distance travelled by A to be

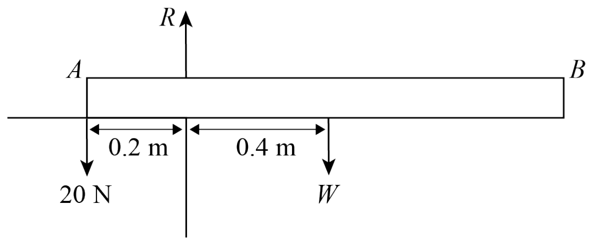
$$1 + \frac{\left(15-\sqrt{3}\right)}{3\left(5+\sqrt{3}\right)}$$
$$= 1.66 \text{ m (3 s.f.)}$$

17 The rod is on the point of tipping so the edge of the table is acting as a pivot and the reaction force is acting vertically upwards at that point. Taking moments about the edge of the table,

$20 \times 0.2 = 0.4W$

$W = \dfrac{20 \times 0.2}{0.4}$

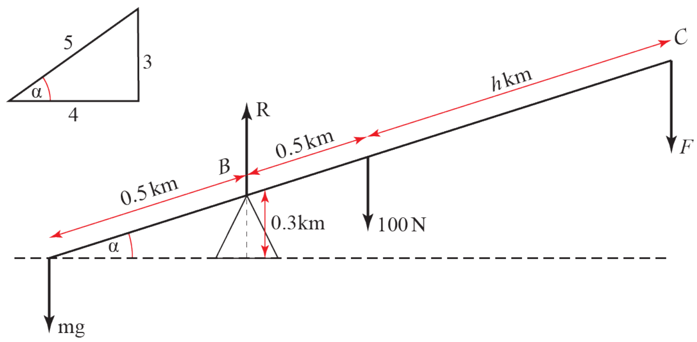
$= 10$ N as required



Challenge

1 The rod makes an angle of $\alpha°$ with the horizontal where

$\sin\alpha = \dfrac{0.3}{0.5} = \dfrac{3}{5} \Rightarrow \cos\alpha = \dfrac{4}{5}$



To lift the mass, total clockwise moments about B must exceed total anticlockwise moments about B:

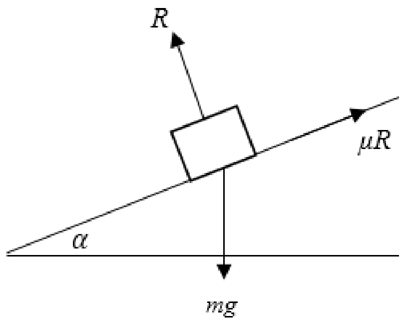
$$(F \times 1.5k\cos\alpha) + (100 \times 0.5k\cos\alpha) > mg \times 0.5k\cos\alpha$$

$$1.5F + 50 > 0.5mg$$

$$\frac{3}{2}F > \frac{1}{2}mg - 50$$

$$F > \frac{1}{3}(mg - 100) \quad \text{as required.}$$

Challenge 2



R(↖) $R = mg\cos\alpha$

(↙) Newton's 2nd law, $F = ma$

$ma = mg\sin\alpha - \mu R$

Therefore

$ma = mg\sin\alpha - \mu mg\cos\alpha$

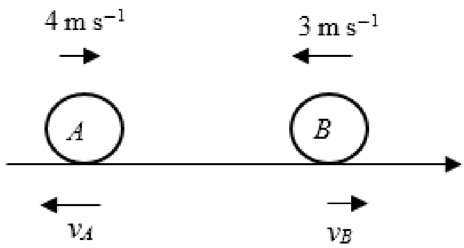
$a = g(\sin\alpha - \mu\cos\alpha)$

Using $s = ut + \frac{1}{2}at^2$ with $u = 0$ and $a = g(\sin\alpha - \mu\cos\alpha)$ gives

$$s = \frac{1}{2}gt^2(\sin\alpha - \mu\cos\alpha) \text{ as required}$$

Challenge 3

Let A be moving in the positive direction and let both balls have a mass of m.
By the conservation of momentum,



momentum before = momentum after

$4m - 3m = -mv_A + mv_B$
$v_B - v_A = 1$

To find the speed of B after the collision use $v = \frac{s}{t}$

$$v_A = \frac{5}{10} = 0.5$$

Since $v_B - v_A = 1$
$v_B = 1.5 \text{ m s}^{-1}$

Practice exam paper

1 a P has speed 0.2 m s^{-1} and Q has speed 0.1 m s^{-1}.
So the distance between P and Q is reducing at a rate of 0.1 m s^{-1}.
As the particles were initially 0.8 m apart it takes
$0.8 \div 0.1 = 8$ s for them to collide.
Alternative method:
Giving distances from initial position of P.
For P: distance = $0.2t$
For Q: distance = $0.8 + 0.1t$
So, P and Q collide when $0.2t = 0.8 + 0.1t$.
$0.1t = 0.8$
$t = 8$ seconds

b By the conservation of momentum,
momentum before = momentum after
$4\times0.2+2\times0.1=4\times0.1+2v$
$2v=0.6$
$v = 0.3$ m s^{-1}

2 a i The train starts to move at t_0 then accelerates, with constant acceleration until t_1.

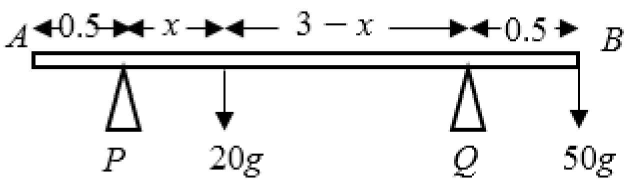
ii The train is travelling with constant velocity.

iii The train decelerates,with constant deceleration, until it is stationary at t_3.

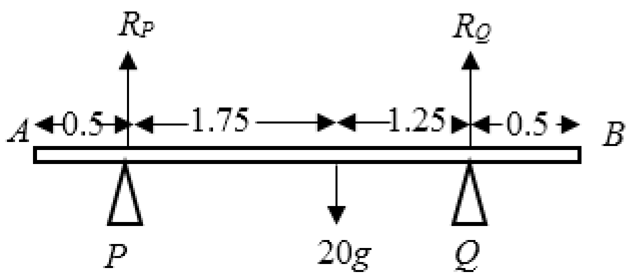
b $80(t_2-t_1)=120$
$t_2-t_1=1.5$
Therefore each unit on the x-axis represents 30 minutes.
So the total length of the journey is 4 hours.

c Total distance travelled d is given by
$d=\frac{1}{2}\times1\times80+120+\frac{1}{2}\times1.5\times80$
$= 220$ km

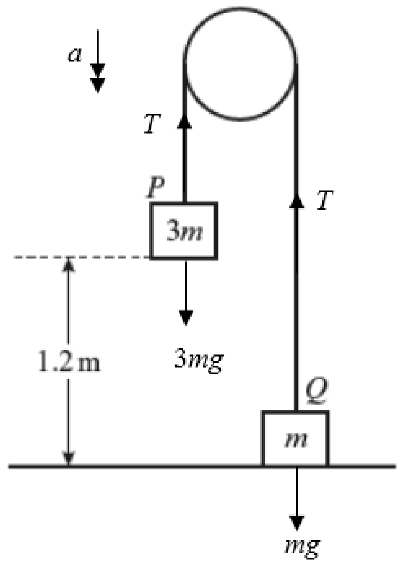
3 a Taking moments about Q
$50g \times 0.5 = 20g \times (3 - x)$
$25 = 20(3 - x)$
$3 - x = 1.25$
$x = 1.75$
Therefore the centre of mass is
$0.5 + 1.75 = 2.25$ m from A



3 b Taking moments about P,
$1.75 \times 20g = 3R_Q$
$R_Q = \frac{35}{3}g$ N
Taking moments about Q,
$1.25 \times 20g = 3R_P$
$R_P = \frac{25}{3}g$ N

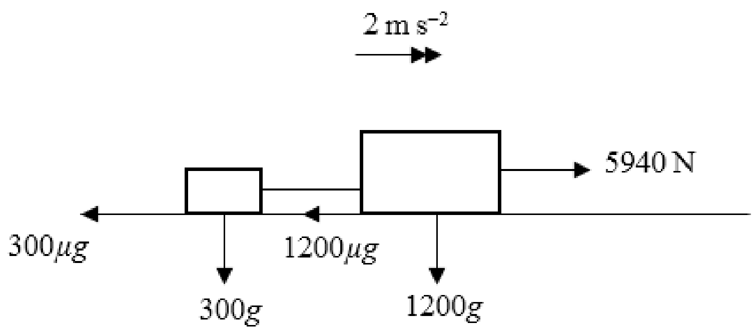


4 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
For the $3m$ kg mass,
$(\downarrow)\ 3mg - T = 3ma$ **(1)**
For the 1 kg mass,
$(\uparrow)\ T - mg = ma$
$T = ma + mg$ **(2)**
Substituting **(2)** into **(1)** gives
$3mg - (ma + mg) = 3ma$
$2mg = 4ma$
$a = 0.5g \text{ m s}^{-2}$



b Using $s = ut + \frac{1}{2}at^2$ gives
$1.2 = \frac{1}{2}(0.5g)t^2$
$t^2 = \frac{2.4}{0.5g}$
$t = \frac{2\sqrt{6}}{7}$
$= 0.700$ s (3 s.f.)

5 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$,
on the system
$(\rightarrow)\ 5940 - 1500\mu g = 1500 \times 2$
$1500\mu g = 5940 - 3000$
$\mu = \frac{5940 - 3000}{1500g}$
$= 0.2$ as required



b For the smaller sledge
$(\rightarrow)\ T - 300\mu g = 300a$
$T = 300a + 300\mu g$
$= 300(2) + 300(0.2)g$
$= 1188$ N

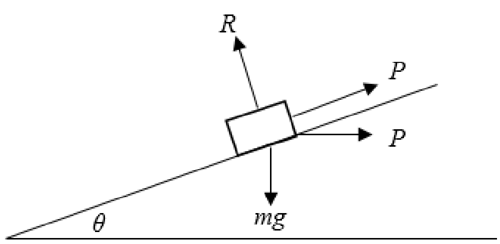
c Constant acceleration/ constant force/ constant friction

d Using $v = u + at$ at point the bar snaps gives
$v = 2(5) = 10\text{ m s}^{-1}$
Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
$(\rightarrow)\ -300\mu g = 300a$
$a = -\mu g$
$\quad = -0.2g\text{ m s}^{-2}$
Using $v^2 = u^2 + 2as$ gives
$0 = 10^2 + 2(-0.2g)s$
$$s = \frac{100}{0.4g}$$
$$= 25.5\text{ m (3 s.f.)}$$

6 a Between A and B
$|\mathbf{v}| = \sqrt{5^2 + 4^2}$
$\quad = \sqrt{41}\text{ km h}^{-1}$
Between B and C
$|\mathbf{v}| = \sqrt{8^2 + (-2)^2}$
$\quad = \sqrt{68}\text{ km h}^{-1}$
Total distance travelled is
$(3\sqrt{41} + 4\sqrt{68})\text{ km}$
Since the ship travels this distance in 7 hours, the average speed between A and C is
$$\frac{(3\sqrt{41} + 4\sqrt{68})}{7} = 7.46\text{ km h}^{-1}\text{ (3 s.f.)}$$

b Let port A be the origin, then the position vector of port C is
$\mathbf{r} = 3(5\mathbf{i} + 4\mathbf{j}) + 4(8\mathbf{i} - 2\mathbf{j})$
$\quad = 47\mathbf{i} + 4\mathbf{j}$
$|\mathbf{r}| = \sqrt{47^2 + 4^2}$
$\quad = \sqrt{2225}$
$\quad = 5\sqrt{89}\text{ km}$
Since the ship is travelling at 10 km h^{-1}, the time taken for the ship to reach A is
$$\frac{5\sqrt{89}}{10} = 4.72\text{ hours (3 s.f.)}$$

7 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
$(\nearrow)\ P + P\cos\theta = mg\sin\theta$
$P(1 + \cos\theta) = mg\sin\theta$
$$P = \frac{mg\sin\theta}{1 + \cos\theta}\text{ as required}$$



7 b Using Newton's second law, $\mathbf{F} = m\mathbf{a}$

(↖) $R = P\sin\theta + mg\cos\theta$

$$R = \frac{mg\sin^2\theta}{1+\cos\theta} + mg\cos\theta$$
$$= \frac{mg\sin^2\theta + mg\cos\theta(1+\cos\theta)}{1+\cos\theta}$$
$$= \frac{mg\sin^2\theta + mg\cos\theta + mg\cos^2\theta}{1+\cos\theta}$$
$$= \frac{mg + mg\cos\theta}{1+\cos\theta}$$
$$= \frac{mg(1+\cos\theta)}{1+\cos\theta}$$

$= mg$ as required

c Using Newton's second law, $\mathbf{F} = m\mathbf{a}$

(↙) $mg\sin 30 - 0.25mg = ma$

$a = 0.5g - 0.25g$

$a = 0.25g$

Therefore initial acceleration is $0.25g\text{ m s}^{-2}$ down the slope.